CSC 125 - Discrete Math I, Spring 2017

Graphs
Definition: A graph $G = (V, E)$ consists of a nonempty set $V$ of vertices (or nodes) and a set $E$ of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Example: This is a graph with four vertices and five edges.
Some Terminology

- In a *simple graph* each edge connects two different vertices and no two edges connect the same pair of vertices.
- A *multigraph* may have multiple edges connecting the same two vertices. When $m$ different edges connect the vertices $u$ and $v$, we say that \{u, v\} is an edge with *multiplicity* $m$.
- An edge that connects a vertex to itself is called a *loop*.
- A *pseudograph* may include loops, as well as multiple edges connecting the same pair of vertices.
Definition: An directed graph (or digraph) $G = (V, E)$ consists of a nonempty set of $V$ vertices (or nodes) and a set of directed edges (or arcs). Each edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair $(u, v)$ is said to start at $u$ and end at $v$. 
Some Terminology (continued)

- A simple directed graph has no loops and no multiple edges.
- A directed multigraph may have multiple directed edges. When there are \( m \) directed edges from the vertex \( u \) to the vertex \( v \), we say that \( (u, v) \) is an edge of multiplicity \( m \).
## Summary of Graph Terminology

<table>
<thead>
<tr>
<th>Type</th>
<th>Edges</th>
<th>Multiple Edges</th>
<th>Loops</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple graph</td>
<td>Undirected</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Multigraph</td>
<td>Undirected</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Pseudograph</td>
<td>Undirected</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Simple directed graph</td>
<td>Directed</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Directed multigraph</td>
<td>Directed</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Mixed graph</td>
<td>Both</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Definition: An adjacency list can be used to represent a graph with no multiple edges by specifying the vertices that are adjacent to each vertex in the graph.
**Definition:** Suppose that $G = (V, E)$ is a simple graph where $|V| = n$. Arbitrarily list the vertices of $G$ as $v_1, v_2, \ldots, v_n$. The *adjacency matrix* $A_G$ of $G$, with respect to the listing of vertices, is the $n \times n$ zero-one matrix with 1 as its $(i, j)th$ entry when $v_i$ and $v_j$ are adjacent, and 0 as its $(i, j)th$ entry when they are not adjacent.
Adjacency Matrices (continued)

- Adjacency matrices can also be used to represent graphs with loops and multiple edges.
- A loop at the vertex $v_i$ is represented by a 1 at the $(i, j)th$ position of the matrix.
- When multiple edges connect the same pair of vertices $v_i$ and $v_j$, the $(i, j)th$ entry equals the number of edges connecting the pair of vertices.
Adjacency matrices can also be used to represent directed graphs. The matrix for a directed graph $G = (V, E)$ has a 1 in its $(i, j)th$ position if there is an edge from $v_i$ to $v_j$ where $v_1, v_2, \ldots, v_n$ is a list of vertices.

The adjacency matrix for a directed graph does not have to be symmetric because there may not be an edge from $v_i$ to $v_j$ when there is an edge from $v_j$ to $v_i$.

To represent directed multigraphs, the value of $a_{ij}$ is the number of edges connecting $v_i$ to $v_j$. 